Arithmetical Word Problems in Russia

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Published in a slightly different version under the title *Ryska skolproblem* in *Svenska Matematikersamfundet Medlemsutskicket*, October 1, 2005, pp. 8 - 11.

In every society certain skills are expected of everybody. For example, if a Brazilian boy tells his mates that he cannot play soccer, he will be considered a nut. Mastery is expected from everyone, so everyone gets it.

Something similar takes place in Russia with respect to word problems. Presence, even abundance of word problems has been normal in Russian school for many decades. The difficulty of problems continually grows from one grade to another, then to olympiads, then to research. Here are a few short statements under which most Russians would subscribe:

• It is good for children to solve multi-step word problems already in elementary school, before starting algebra,.

• Children are motivated to solve arithmetical word problems because they are generally motivated to overcome difficulties, both physical and intellectual, and are willing to train themselves for that if the tasks are within their possibilities and the society approves it.

• Solving word problems is a good opportunity for the children to display and train creativity. The fact that the answer is unique and predetermined by the data by no means contradicts this.

• If you can solve a word problem without algebra, it is good for you. Generally, the more bare-handed you solve a problem, the better for you.

• It is normal, even necessary that teachers require correct, clear and explicit solutions and answers.

• The younger are children, the less they should be allowed to decide what to study. Solving arithmetical word problems in elementary school should be obligatory for every healthy child.

• The government should plan the minimal version of school studies and state the minimal level of difficulty of problems solved in every grade.

• Word problems do not need to be realistic in the literal sense. They are solved for the sake of general intellectual development rather than for a literal application to everyday life.

The following small selection contains arithmetical word problems of different levels taken from Russian sources with short comments.

The following problem is from a recent Russian textbook for the 2-d grade:

Problem 1. Vintik and Shpuntik agreed to go to the fifth car of a train. However, Vintik went to the fifth car from the beginning, but Shpuntik went to the fifth car from the end. How many cars has the train if the two friends got to one and the same car? [Geidman.2.1, p.9].

The following problem is from a well-known russian book for children written by Nosov. Its main character Vitya Maleev did poorly in mathematics in the third grade and promised his teacher to train himself in solving problems from the 3-grade textbook. This is one of them.

Problem 2. A boy and a girl collected 24 nuts. The boy collected twice as many nuts as the girl. How many did each collect?

In Russia problems like this one are called *problems by parts*, because they can be solved using the notion of a *part*, which prepares children to use a letter to denote an unknown quantity. The author describes Vitya's process of thinking in much detail. First Vitya divides 24 by two and gets 12. May be, each collected 12 nuts? No, the boy collected more than the girl. Not knowing what to do, Vitya draws a picture of a boy and a girl. To express the fact that the boy collected two times more nuts than the girl, Vitya draws two pockets on the boy's pants and one pocket on the girl's apron. Then he looks at his picture and sees *three pockets*. Then an idea "like a lightning" comes to his mind: he should divide the number of nuts into the number of pockets! Thus he gets $24 : 3 = 8$. So each pocket contains 8 nuts. This is how many nuts the girl has. The boy has two pockets, so he has 8 times 2, which gives 16. Now Vitya can check his answer: he adds 8 and 16 and gets 24. Now he is sure that his solution is correct. He is very excited. Probably, this is the first time in his life that he solved a problem on his own. He goes to the street to tell somebody about it. A neighbor girl says: "This is a third-grade problem. We solved them last year." This does not diminish Vitya's joy and he is right: he made a discovery.

The following problem is from a recent Russian textbook for the 4-th grade:

Problem 3. When Ivan Tsarevich came to the Magic Kingdom, Koschey was as old as Baba Yaga and Ivan Tsarevich together. How old was Ivan Tsarevich when Koschey was as old as Baba Yaga was when Ivan Tsarevich came to the Magic Kingdom? [Geidman.4.1, p.104].

Ivan Tsarevich, Koschey and Baba Yaga are well-known characters of Russian folk tales. The problem is solvable, although it gives none number!

Problem 4. Deniska can eat a jar of jam in 6 minutes. Mishka can eat a similar jar of jam two times faster. In how much time will they eat a jar of jam together?

[Geidman.4.1, p.34].

Deniska and Mishka are colloquial versions of common Russian names, which fit into the jocular style of this problem. However, this problem is very seminal. It introduces children into the realm of rates.

The following problem appeared first in a book by Perelman, then at a Moscow mathematical olympiad in 1940 and a few years later was included into a school problem book for 5-6 grades:

Problem 5. A boat, going downstream, made a distance between two ports in 6 hours and returned in 8 hours. How much time is needed for a raft to make this distance downstream? [Berez, p.246].

To solve it, we may take the distance in one direction as a unit of length and call it *one way*. Then the velocities of the boat downstream and upstream are *1/6 way/hour* and *1/8 way/hour* respectively. They are different because the current in one case increased and in the other case decreased the boat's velocity. Thus the velocity of the current is half the difference between them, that is

$$\left(\frac{1}{6}-\frac{1}{8}\right)\div 2=\frac{1}{48}\quad way/hour.$$

Velocity of the raft equals velocity of the current. To make one *way* with this velocity, one needs the quantity of hours, which is equal to 1 divided by 1/48, that is 48 hours.

Problem 6. An ancient problem. A flying goose met a flock of geese in the air and said: *"Hello, a hundred geese!"* The leader of the flock answered to him: *"There is not a hundred of us. If there were as many of us as there are and as many more and half as many more and quarter as many more and you, goose, also flied with us, then there would be a hundred of us."* How many geese were there in the flock? [Larichev, p.37].

This problem may be solved in two ways, which is especially useful in introducing algebra. First you may call "one part" the quantity of geese in the flock. Then

$$1 \text{ part } + 1 \text{ part } + \frac{1}{2} \text{ part } + \frac{1}{4} \text{ part } + 1 = 100.$$

Collecting coefficients we get

$$1 + 1 + \frac{1}{2} + \frac{1}{4} = \frac{11}{4}.$$

Thus 11/4 parts equals 100-1=99. Thus one part is 99 divided by 11/4, which is 36. This is halfway to algebra. Writing X instead of "part", we get an algebraic equation

$$X + X + \frac{1}{2}X + \frac{1}{4}X + 1 = 100,$$

which can be solved by algebraic means.

One very important quality of word problems is that they connect arithmetics and algebra with physics and geometry. Let us present two examples of this.

Problem 7. A HISTORICAL PROBLEM. A swimmer was swimming upstream Neva River. Near the Republican Bridge he lost an empty flask. After swimming 20 min more upstream he noticed his loss and swam back to find the flask; he reached it near the Leughtenant Schmidt Bridge. Find the velocity of current at Neva River if the distance between these two bridges is 2 km. [Larichev, p.208].

This problem shows the power of the physical idea of relative movement. Let us place ourselves in the coordinate system moving with the stream. In this system the flask does not move when it is lost, while the swimmer swims first away from it, then towards it. His proper velocity is assumed constant, so he spends one and the same time swimming in both directions. But one of them took 20 minutes, so the other also takes 20 minutes, so the total time when the flask was lost is 40 minutes. Now we return to the coordinate system where we were before, and notice that the flask took 40 minutes to move from one bridge to the other, that is 2 km. So the velocity of the current is 2 km divided by 2/3 hour, which makes 3 km/h.

The following is a (slightly changed) word problem with geometric content, included in one of Perelman's books:

Problem 8. A man sold firewood. To make standard portions, he always used one and the same rope, surrounded a pack of logs with it and brought it into houses on his back. One woman asked him to bring a double portion of firewood. The man proceeded as usual, only took a rope one and a half times longer than usual. The woman complained: *"Since I paid you a double fee, you should use a double length rope."* The man replied: *"You are mistaken, mam. In fact, I brought you even a little more firewood than you requested."* Who is right? [Perelman, p.27].

To solve this problem, we have to make simplifying assumptions, as it is usual in applied science. We assume that the firewood surrounded by a rope is a cylinder, whose height is the length of the logs and base's circumference equals the lenght of the rope. Since the height of the cylinder is constant, the volume of the cylinder is proportional to the area of the base, which is proportional to the square of the radius or, which is the same, to the square of the circumference. So, if the length of the rope is multiplied by 3/2, the amount of the firewood is multiplied by a square of this amount, which is 9/4, which is really a little more than 2. The man was right.

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