RECEPTOR ÓTIMO PARA CÓDIGOS DE BLOCO USANDO DECISÃO SUAVE

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Um receptor para códigos de bloco é apresentado o qual maximiza a probabilidade a posteriori das palavras código, em um canal discreto no tempo e sem memória, quando as palavras código são equiprováveis. Quando são considerados códigos lineares, o uso da treliça associada na decodificação torna prático o emprego deste receptor por simplificar o processo de decodificação, ao mesmo tempo preservando a otimalidade do algoritmo.

Seja $c = (c_0, c_1, \ldots, c_{n-1})$ uma palavra qualquer de um código de comprimento n, contendo k dígitos de informação. Os elementos c_i pertencem a um campo de Galois $GF(p)$. O receptor opera na sequência recebida $r = (r_0, r_1, \ldots, r_{n-1})$, onde r_j é um número real, decidindo pela palavra c que maximiza a seguinte expressão

$$P(c/r) = \prod_{l=0}^{n-1} \sum_{i=0}^{p-1} P(r_l/i) \sum_{j=0}^{p-1} \theta^{j(c_l - i)}$$

onde $\theta = \exp(2\pi\sqrt{-1}/p)$ e $P(c/r)$ representa a probabilidade a posteriori das palavras código. Este é o resultado fundamental deste artigo e que, quando aplicado a códigos binários, assume a seguinte forma:

$$\ln P(c/r) = \ln\lambda + \sum_{l=0}^{n-1} c_l \ln\phi_l$$

onde λ independe das palavras c e ϕ_l é a razão de verossimilhança. Isto significa que, no caso binário, a maximização de P(c/r) é conseguida escolhendo-se a palavra c que maximiza $\sum_{l=0}^{n-1} c_l \ln\phi_l$.

Resultados da simulação para avaliação do desempenho deste receptor e comparação com procedimentos convencionais são apresentados.

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1. INTRODUCTION

An optimum soft-decision receiver is introduced for decoding the output of a noisy multilevel communication system operating with equally likely codewords.

The codewords are transmitted through a time-discrete memoryless channel characterized by the likelihood function $P(r_j|i)$, $i\epsilon\{0,1,\ldots,p\}$, $r_j \epsilon R$. Assume the codewords are transmitted as blocks of n digits.

Once a word is transmitted and the sequence $r = (r_0, r_1, \ldots, r_{n-1})$ is received the use of the classical procedure of selecting a codeword which maximizes the a posteriori probability minimizes the overall probability of error when the codewords are equiprobable [1]. However except for a few classical channels it is not normally very clear how one should process the received sequence in order to maximize the a posteriori probability.

In what follows a procedure is established for achieving this optimum processing of the received sequence. If linear codes are employed then a considerable simplification results from the use of a trellis for decoding [2].

2. THE RECEIVER

Let $c = (c_0, c_1, \ldots, c_{n-1})$ represent any codeword of a block code of length n containing k information digits, i.e. a (n,k) code. The code symbols are taken from the Galois field $GF(p)$. The receiver operates on the received sequence $r = (r_0, r_1, \ldots, r_{n-1})$, $r_j \epsilon R$ as follows. Decide for the codeword c which maximizes the expression

$$\prod_{\ell=0}^{n-1} \sum_{i=0}^{p-1} P(r_\ell|i) \sum_{j=0}^{p-1} \theta^{j(c_\ell - i)} \qquad (1)$$

where $\theta = \exp[2\pi\sqrt{-1}/p]$. *Next it is proved that the codeword which maximizes* (1) *also maximizes the a posteriori probability* $P(c|r)$. *From probability theory it is known that*

$$P(c|r) = P(c).P(r|c)/P(r) \qquad (2)$$

However if the codewords are equiprobable $P(c) = p^{-k}$ *and* (2) *can be written as*

$$P(c|r) = p^{-k}\, P(r|c)/P(r) \qquad (3)$$

In terms of a finite Fourier transform [3] (3) *can be written as*

$$P(r|c) = p^{-n} \sum_{u \in V_n} F(r,u)\, \theta^{u.c} \qquad (4)$$

$$F(r,u) = \sum_{v \in V_n} P(r|v)\, \theta^{-u.v} \qquad (5)$$

In (4) *and* (5) u *and* v *are elements of* V_n, *an* n *dimensional vector space over* $GF(p)$ *and* $u.v$ *denotes their inner product. Because the channel is memoryless* (5) *becomes*

$$F(r,u) = \sum_{v \in V_n} \prod_{\ell=0}^{n-1} P(r_\ell|v_\ell)\, \theta^{-u_\ell . v_\ell} = \prod_{\ell=0}^{n-1} \sum_{i=0}^{p-1} P(r_\ell|i)\, \theta^{-iu_\ell} \qquad (6)$$

Substitution of (4) *and* (6) *in* (3) *gives*

$$P(c|r) = [p^{-k-n}/P(r)] \sum_{u \in V_n} F(r,u)\, \theta^{u.c}$$

$$= [p^{-k-n}/P(r)] \sum_{u \in V_n} \prod_{\ell=0}^{n-1} \sum_{i=0}^{p-1} P(r_\ell|i)\, \theta^{u_\ell (c_\ell - i)}$$

$$= [p^{-k-n}/P(r)] \prod_{\ell=0}^{n-1} \sum_{i=0}^{p-1} P(r_\ell|i) \sum_{j=0}^{p-1} \theta^{j(c_\ell - i)} \qquad (7)$$

This completes the proof that the assertion made in conjunction with (1) maximizes the a posteriori probability and is therefore optimum.

If one restricts this receiver for operation with binary alphabets, ie. p=2, (1) *is then considerably simpler. Substituting* p=2 *in* (1) *yields*

$$P(c|r) = [2^{-k-n}/P(r)] \prod_{\ell=0}^{n-1} \sum_{i=0}^{1} P(r_\ell|i) \sum_{j=0}^{1} \theta^{j(c_\ell - i)}$$

$$= [2^{-k-n}/P(r)] \prod_{\ell=0}^{n-1} [P(r_\ell|0)(1+\theta^{c_\ell}) + P(r_\ell|1)(1-\theta^{c_\ell})] \qquad (8)$$

By using the likelihood ratio $\phi_\ell = P(r_\ell|1)/P(r_\ell|0)$ and $\rho_\ell = (1-\phi_\ell)/(1+\phi_\ell)$ *in* (8) *one gets*

$$P(c|r) = [2^{-k-n}/P(r)] \prod_{\ell=0}^{n-1} [1+\theta^{c_\ell}+\phi_\ell(1-\theta^{c_\ell})]P(r_\ell|0)$$

$$= [2^{-k-n}/P(r)] \prod_{\ell=0}^{n-1} [1+\phi_\ell + \theta^{c_\ell}(1-\phi_\ell)]P(r_\ell|0)$$

$$= [2^{-k-n}/P(r)] \prod_{\ell=0}^{n-1} [1+\rho_\ell \theta^{c_\ell}]P(r_\ell|0)(1+\phi_\ell) \qquad (9)$$

However

$$\theta^{c_\ell} = \exp\left[(2\Pi/2)\sqrt{-1}\, c_\ell\right] = \begin{cases} 1 & , \; c_\ell = 0 \\ -1 & , \; c_\ell = 1 \end{cases}$$

That is $\theta^{c_\ell} = (-1)^{c_\ell}$ *which is substituted in (9) to give*

$$P(c|r) = [2^{-k-n}/P(r)] \prod_{\ell=0}^{n-1} [1+\rho_\ell(-1)^{c_\ell}] P(r_\ell|0)(1+\phi_\ell)$$

$$= \beta \prod_{\ell=0}^{n-1} [1+\rho_\ell(-1)^{c_\ell}] \qquad (10)$$

where $\beta = [2^{-k-n}/P(r)] \prod_{\ell=0}^{n-1} P(r_\ell|0)(1+\phi_\ell)$ *does not depend on* c. *Still, further simplification is obtained by observing that*

$$1 + \rho_\ell(-1)^{c_\ell} = 2\phi_\ell^{c_\ell}/(1+\phi_\ell) \qquad (11)$$

So, taking (11) into (10) one finally gets

$$P(c|r) = [2^{-k}/P(r)] \prod_{\ell=0}^{n-1} P(r_\ell|0) \prod_{\ell=0}^{n-1} \phi_\ell^{c_\ell} = \lambda \prod_{\ell=0}^{n-1} \phi_\ell^{c_\ell} \qquad (12)$$

where $\lambda = [2^{-k}/P(r)] \prod_{\ell=0}^{n-1} P(r_\ell|0)$ *does not vary with* c. *By taking the natural logarithm of both sides in (12) one obtains*

$$\ell n\, P(c|r) = \ell n \lambda + \sum_{\ell=0}^{n-1} c_\ell\, \ell n\, \phi_\ell \qquad (13)$$

This means that the maximization of $P(c|r)$ *in the binary case is achieved by choosing the codeword* $c = (c_0, c_1, \ldots, c_{n-1})$ *which maximizes* $\sum_{\ell=0}^{n-1} c_\ell\, \ell n\, \phi_\ell$. *This last result is interesting for its simplicity and compactness. Also, for the particular case of a channel disturbed by additive white gaussian noise a correlation receiver results as was to be expected.*

3. APPLICATION TO LINEAR BLOCK CODES

As an example let us consider the reception of words which are made up of on-off RF pulses, disturbed by additive white gaussian noise, using an envelope detector. It is assumed that the envelope of the received waveform is sampled at intervals far enough apart so that it is reasonable to suppose the samples to be statistically independent.

The signal input to the envelope detector is

$$z(t) = S_i(t) + n(t) \quad , \quad 0 \le t \le T \, , \quad i \in \{0,1\}$$
$$S_0(t) = 0$$

This expression can be written as a narrowband signal, thus $z(t) = A_i \cos\omega_0 t + x_c(t)\cos\omega_0 t - x_s(t)\mathrm{sen}\,\omega_0 t$ where $A_0 = 0$, A_1 is a constant and x_{ct} and x_{st} are gaussian random variables [4].

The output $r(t)$ of the envelope detector is given by

$$r(t) = \{\left[A_i + x_c(t)\right]^2 + x_s^2(t)\}^{1/2} \quad , \quad i \in \{0,1\}$$

In the absence of a signal, ie. when $A_0 = 0$ is present, the probability density of the envelope at time ℓ is given by [5]

$$p_0(r_\ell) = \frac{r_\ell}{\sigma^2} \exp(-r_\ell^2/2\sigma^2) \quad , \quad r_\ell \ge 0$$
$$= 0 \quad , \quad r_\ell < 0$$

where σ^2 is the noise power. When the signal is present the probability density of the envelope at time ℓ is given by [5]

$$p_1(r_\ell) = \frac{r_\ell}{\sigma^2} \exp\left(-\frac{r_\ell^2 + A_1^2}{2\sigma^2}\right) I_0\left(\frac{A_1 \, r_\ell}{\sigma^2}\right), \quad r_\ell \ge 0$$
$$= 0 \quad , \quad r_\ell < 0$$

The likelihood ratio is given by

$$\phi_\ell = \frac{p_1(r_\ell)}{p_0(r_\ell)} = \exp(-A_1^2/2\sigma^2) I_0(r_\ell A_1/\sigma^2), \quad r_\ell \ge 0$$

For low signal-to-noise ratios it is almost always true that

$$I_0(r_\ell A_1/\sigma^2) = \exp\left(\frac{r_\ell^2 \, A_1^2}{4\sigma^2}\right)$$

where $I_0(x)$ is the modified Bessel function of first kind and zeroth order [5].

Therefore

$$\phi_\ell = \exp\left[\frac{-A_1^2}{2\sigma^2} + \frac{r_\ell^2 A_1^2}{4\sigma^2}\right]$$

or

$$\ln \phi_\ell = \frac{A_1^2}{2\sigma^2}\left[\frac{r_\ell^2}{2\sigma^2} - 1\right]$$

As the r_ℓ are nearly independent we obtain, with the aid of (13), the following decoding rule.

Choose the codeword which maximizes the expression

$$\sum_{\ell=0}^{n-1} c_\ell \left[\frac{r_\ell^2}{2\sigma^2} - 1\right] \tag{14}$$

Suppose the received n-tuple is r=(1.7,.1,.5) and that a linear single parity check code of length n=3 is the code employed in the above context. The use of the trellis diagram [2] of figure 1 below is very convenient in finding the codeword that maximizes (14). The decoding steps are illustrated in figure 2. We are assuming the gaussian noise to have zero mean and unit variance.

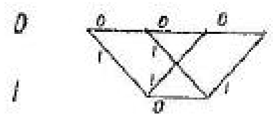

Figure 1. Trellis for the (3,2) Code.

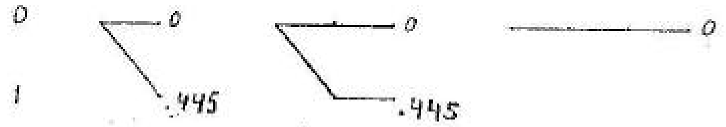

Figure 2. Decoding Steps for the (3,2) Code.

It can be seen that an error has been corrected in the first digit since the hard decision estimate of the transmitted codeword is (1,0,0), because r_ℓ is compared with the threshold $\sqrt{2}\,\sigma$ [5], and our decoded output is (0,0,0). Observe that the correction was made in the position whose sample (r_0=1.7) was nearest to the threshold ie. the leastreliable position.

4. CONCLUSIONS

An optimum soft-decision receiver for multilevel block codes has been introduced. A considerably simpler expression for finding the most likely codeword results when the code symbols are binary. This expression can be thought of as some generalized form of correlation,ie. the receiver correlates the codewords with a vector whose components are $\ln\phi_\ell$ and chooses the codeword that gave the highest correlation. If the code used is linear then the correlations involved are simplified by the use of a trellis for decoding, especially for high rate codes. The complexity of this decoding rule is comparable to those of Hartmann & Rudolph [3] and Wolfenson & Rocha [6]. Finally, if in the example given the high signal-to-noise ratio case is considered, the results obtained are very similar to those of synchronously detected signals disturbed by additive white gaussian noise.

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