ORTHOGONAL MULTILEVEL SPREADING SEQUENCE DESIGN

H.M. de Oliveira, R.M. Campello de Souza
CODEC- Grupo de Pesquisas em Comunicações
Departamento de Eletrônica e Sistemas - CTG- UFPE
C.P. 7800, 50711-970, Recife-PE, Brazil
E-mail: {hmo,ricardo}@npd.ufpe.br

Abstract

Finite field transforms are offered as a new tool of spreading sequence design. This approach exploits orthogonality properties of synchronous nonbinary sequences defined over a complex finite field. It is promising for cellular mobile communications and channels supporting a high signal-to-noise ratio. New digital multiplex schemes based on such sequences have also been introduced which are multilevel Code Division Multiplex. These multiplex schemes termed Galois-field Division Multiplex (GDM) are based on transforms for which there exists fast algorithms. A Digital Signal Processor (DSP) can easily carry out their implementation. A new Efficient-bandwidth code-division-multiple-access (CDMA) is introduced which is based on multilevel spread spectrum sequences over a Galois field. It presents better spectral efficiency (bits/s/Hz) than classical multiple access digital schemes.

Keywords- Spread sequences design, Code-division multiple access, Finite field transforms.

1. Introduction

Digital multiplex usually concerns Time Division Multiplex (TDM). However, it can also be achieved by Coding Division Multiplex (CDM) which has recently been the focus of interest, especially after the IS-95 standardisation of the CDMA system for cellular telephone [QUAL 92]. The CDMA is becoming the most popular multiple access schemes for mobile communication. Classical multiplex increases simultaneously the transmission rate and the bandwidth by the same factor, keeping thus the spectral efficiency unchanged. In order to achieve (slight) better spectral efficiencies, classical CDMA uses waveforms presenting a nonzero but residual correlation. We introduce here a new and powerful issue on CDMA techniques that can be implemented by fast transform algorithms.

In this paper a design of spreading spectrum sequences is introduced which is based on the Finite Field Fourier Transform (FFFT) [POL 71]. We also consider the recently introduced finite field Hartley transform (FFHT) [CAM *et al.* 98]. Alike classical Galois-Fourier transforms [BLA 79], the FFHT, which is defined on a Gaussian integer set $GI(p^m)$ contains some redundancy and only the cyclotomic coset leaders of the transform coefficients need to be transmitted. This yields new "*Efficient-Spread-Spectrum Sequences for band-limited channels*". Trade-offs between the alphabet extension and the bandwidth are exploited in the sequel. Another point to mention is that the superiority of the spreading spectrum sequences is essentially due to the their low implementation complexity.

2. New Synchronous Multilevel Spreading Spectrum Sequences

Given a signal v over a finite field GF(p), we deal with the Galois domain considering the spectrum V over an extension field $GF(p^m)$ which corresponds to the Finite Field Transform (Galois Transform) of the signal v [BLA 79]. Let $v = (v_0 , v_1 , ... , v_{N-1})$ be a vector of length N with components over GF(q), $q = p^r$. The FFFT of v is the vector $F = (F_0 , F_1 , ..., F_{N-1})$ of components $F_k \in GF(q^m)$, given by

$$F_k := \sum_{i=0}^{N-1} v_i \alpha^{ki} ,$$

where α is a specified element of multiplicative order N in $GF(q^m)$.

Each symbol in the ground field GF(p) has duration T seconds. Spreading waveforms can be used to implement an N-user multiplex on the extension field $GF(p^m)$ where $N \mid p^m-1$. A (p-1)-CDM considers digital carrier sequences per channel as versions of the cas(.) function over the Galois (complex) field GI(p). The cyclic digital carrier has the same duration T of an input modulation symbol, so that it carries N slots per data symbol. The interval of each cas-symbol is T/N and therefore the bandwidth expansion factor when multiplexing N channels may be roughly N, the same result as FDM and TDM/PAM.

A spread spectrum multiplex is showed in the figure 1: The output corresponds exactly to the Galois-Hartley Transform of the "user"-vector $(v_0,v_1,...,v_{N-1})$. Therefore, it contains all the information about all channels. Each coefficient V_k of the spectrum has duration T/N.

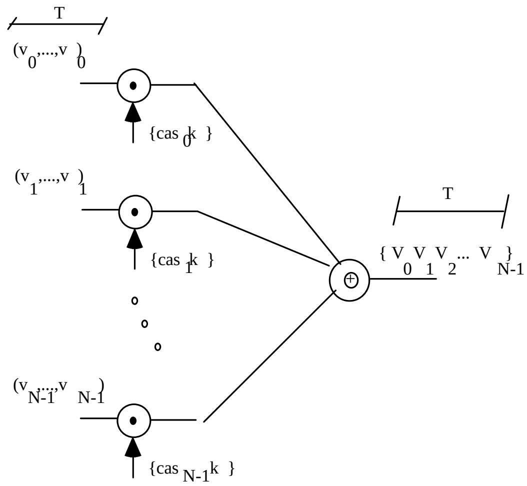

Figure 1. Galois-Field MUX: Spreading sequences.

These carriers can be viewed as spreading waveforms [MAS 95]. An N-user mux has N spreading sequences, one per channel. The requirements to achieve Welch's lower bound according to Massey and Mittelholzer [MAS&MIT 91], are achieved by these sequences.

3. Interpreting Galois-Hartley Transform over GF(5) as spreading waveforms

A 4-channel complex spreading waveforms over GF(5) can be chosen as:

$\{cas_0k\}= \{1, 1, 1, 1\}$ $\{cas_1k\}= \{1, j3, 4, j2\}$
$\{cas_2k\}= \{1, 4, 1, 4\}$ $\{cas_3k\}= \{1, j2, 4, j3\}$.

The digital carriers are defined on a complex Galois field GI(p) where the element may or not may belong to GF(p), although the original definition [CAM et al. 98] considers -1 as a quadratic non-residue in GF(p). Two distinct cases are to be considered: p=4k+1 and p=4k+3, k integer. For instance, considering $j\in GF(5)$ then $2^2\equiv -1$ (mod 5) so $j=\sqrt{-1}\equiv 2$ (mod 5). Two-dimensional digital carriers then degenerated to one-dimensional carriers.

In the absence of noise, there is no cross talk from any user to any other one, which corresponds to orthogonal carrier case. Considering the above example, carriers are reduced to Hadamard carriers. Therefore, this spreading sequences can be interpreted as some kind of generalisation (multilevel and 2-dimensional) of classical synchronous orthogonal Hadamard spreading spectrum sequences.

4. Implementation of a Spreading Spectrum scheme by Finite Field Transforms

The large success achieved by Ungerboeck's coded-modulation came from adding redundancy by an alphabet expansion [UNG 82]. In classical channel coding, redundant signals are appended to information symbols in a way somewhat analogous to time division multiplex. In the present work, the coded-modulation idea is adapted to multiplex: Information streaming from users are not combined by interleaving (like TDM) but rather by a signal alphabet expansion. The multiplex of users' sources over a Galois Field GF(p) deals with an expanded signal set having symbols from an extension field $GF(p^m)$, m>1. The new multiplex is carried out over the Galois domain instead of the Frequency or time domain. As an attractive implementation, the multiplex can be carried out by a Galois Field Transform (FFFT/FFHT) so the DEMUX corresponds exactly to an Inverse Finite-Field Transform of length $N \mid p^m-1$. Transform-based multiplexes by spreading spectrum perform as follows.

First, the Galois spectrum of N-user GF(p)-signals is evaluated. The spectral compression is achieved by eliminating the redundancy: only the leaders of cyclotomic cosets are transmitted. The demultiplex is carried out (after signal regeneration) recovering the complete spectrum by "filling" missing components from the received coset leaders. Then, the inverse finite field transform is computed to obtain the user's signals.

Suppose that users data are p-ary symbols transmitted at a speed B=1/T bauds. Let us consider the problem of multiplexing N users. The number of cyclotomic cosets associated with a finite field spectrum is denoted n. The clock driving multilevel (finite field) symbols is N/n times faster than the input baud rate. The bandwidth requirements will be (N/n).B instead of NB.

5. References

[BLA 79] R.E. Blahut, Transform Techniques for Error Control Codes, *IBM J. Res. Develop*, 23, n.3, pp. 299-314, May, 1979.

[CAM *et al.* 98] R.M. Campello de Souza, H.M. de Oliveira, A.N. Kauffman and A.J.A. Paschoal, "Trigonometry in Finite Fields and a new Hartley Transform", *IEEE International Symposium on Information Theory*, ISIT, MIT Cambridge, MA, THB4: Finite Fields and Appl., p. 293, 1998.
(see http://lids.mit.edu/ISIT98)

[MAS 95] J.L. Massey, *Towards an Information Theory of Spread-Spectrum Systems, in: Code Division Multiple Access Communications*, Eds S.G. Glisic and P.A. Leppnen, Boston, Dordrecht and London, Kluwer, pp. 29-46, 1995.

[MAS&MIT 91] J.L. Massey and T. Mittelholzer, Welch's bound and sequence sets for Code-Division-Multiple-Access Systems, *Proc. of Sequences'91*, Springer-Verlag, 1991.

[POL 71] J.M. Pollard, The Fast Fourier Transform in a Finite Field, *Math. Comput.*, 25, pp. 365-374, Apr., 1971.

[QUAL 92] Qualcomm, *The CDMA Network Engineering Handbook*, Qualcomm Inc., San Diego, CA, 1992.

[UNG 82] G. Ungerboeck, Channel Coding with multilevel/phase signals, *IEEE Trans. Info. Theory*, IT 28, pp. 55-67, Jan., 1982.