Adaptative Repetition Coding as a Pseudorandom Issue for WiMedia UWB-Payload Protection

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Abstract— **This article describes an encoding strategy through repetition codes for error protecting of the WiMedia payload. Such codes have usually their application rather restrained due to their low transmission rates over low error probabilities. A binary symmetric channel (BSC) was used to evaluate the system performance and a number of simulations were done to assess the influence of the parameters of the coded system. The chief idea is to select the maximum acceptable error probability and fade probability followed by the designing of the repetition coding scheme so as to maximize the transmission rate. Simulations results show that this pseudorandom adaptive technique presents better performance at low error probability, besides offering a tradeoff between rate and error probability.**

Keywords— repetition codes; WiMedia UWB; BSC channel; transmission rate

I. INTRODUCTION

The digital transmission has as its greatest strengths the ability to guarantee projected reliability –being reliability expressed through a straightforward and easy to evaluate parameter: the error probability. One of the most celebrated results of the Information Theory [1] is Shannon´s random coding argument [2]. It establishes that long random codes are good, in the sense of error control. Now, there is a natural unbalance between coding and decoding: The main problem of practical coding schemes concerns essentially the decoder complexity [1-3]. The two more elementary error-control techniques are the repetition codes and the employ of symbol erasure, which are both suitable for communication systems that involve automatic request (ARQ) strategies [4]. This couple of tools is precisely the primary ingredient in the new coding schemes introduced here. An ensemble of repetition codes is equipped with a probability distribution, which yield in a simple random scheme. The implementation of the scheme is carried out by a pseudorandom strategy. The decoding of block codes (n,1) is usually performed using majority logic [5] in the cases of hard decision, or by a simple two-states trellis [6, 7], in the cases of soft decision.

WiMedia UWB [8, 9] is an OFDM-based standard developed to operate at high transmission rates (ranging from 53.3 Mbps to 480 Mbps). As presented in [10], the use of repetition codes has advantages in protecting the WiMedia header, and it is additionally suggested the use of repetition codes in the payload error-protection.

Starting with an ensemble of distinct repetition codes with different transmission rates, it is suggested a probabilistic combination of these codes according to a certain probability distribution. This distribution is chosen so as to maximize the channel transmission rate, subject to minimal quality requirements, expressed by an upper bound on the decoding error probability. This approach allows adopting different profiles suitable to pre-set error probabilities, which can be used to select the convenient environment–dependent profile, thereby supplying more flexibility to the WiMedia UWB.

Although the theoretical approach describes a random coding, a naïve trick is used, turning pseudo-random the error-control process and making the decoding possible. This article is organized as follows. Section II presents the design of the coding scheme, and examines the main parameter of the system. Section III shows simulation results, as well as the proposal of inserting adaptive repetition codes in the WiMedia standard, the Section IV deals with this scheme applied to WiMedia UWB. Section V, conclusions and final comments.

II. MATHEMATICAL FORMULATION

This section presents the main idea and steps in the design of a random coding system by adjusting the probabilities of selection of repetition codes with blocks 1 to N, where N represents the blocklength of the longest repetition code. Each of the following subsections shows a formulation based on a BSC channel, chosen for simulations due to the fact that it can be used to represent a great amount of practical channels and because of its simplicity. Nevertheless, no further constraints are made about the channel in the analysis, so the other channels can as well be used. The idea is guarantee a transmission with error probability bounded by a pre-set value P_e.

A. Transmission rate

The transmission rate for an individual block code with parameters $(n,1)$ is given by $R_n=1/n$. In the random model proposed in this paper, there is (possibly) a different repetition code selected for each transmitted bit. Let us define P_n as the

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probability of a repetition code of blocklength *n* be chosen at a particular information bit. Assuming a finite set of repetition codes, where the maximum blocklength is *N*, the average transmission rate E*(1/n)* can be calculated by the following equation:

$$R = E\left(\frac{1}{n}\right) = 1.P_1 + \left(\frac{1}{2}\right).P_2 + \left(\frac{1}{3}\right).P_3 + \cdots + \left(\frac{1}{N}\right).P_N \quad (1)$$

$$R = \sum_{n=1}^{N} \frac{1}{n}.P_n \quad (2)$$

B. Error Probability

The error probability of the repetition coding system (P_ε) depends on the crossover probability of the channel (ε). This actual decoding error probability is selected as a controlled parameter. Now, taking P_n as the probability of selecting a repetition code of blocklength *n*, the probability P_ε is given by:

$$P_\varepsilon = P_1\varepsilon + P_2\varepsilon^2 + P_3[\binom{2}{3}\varepsilon^2(1-\varepsilon) + \binom{3}{3}\varepsilon^3(1-\varepsilon)^0] +$$
$$+P_4[\binom{3}{4}\varepsilon^3(1-\varepsilon) + \binom{4}{4}\varepsilon^4(1-\varepsilon)^0] +$$
$$+P_5[\binom{3}{5}\varepsilon^3(1-\varepsilon)^2 + \binom{4}{5}\varepsilon^4(1-\varepsilon) + \binom{5}{5}\varepsilon^5(1-\varepsilon)^0] +$$
$$+\cdots \quad (3)$$

which gives in a compact notation:

$$P_\varepsilon = \sum_{n=1}^{N} P_n \sum_{z=1}^{\frac{n+(n\,mod2)}{2}} \binom{n}{\tau+z} \varepsilon^{\tau+z}(1-\varepsilon)^{\tau-z}, \quad (4)$$

where $\tau = \frac{n-(n\,mod2)}{2}$.

C. Erase Probability

An erase happens in a repetition code scheme when a code with even blocklength is selected, and the amount of bit errors after transmission is exactly half of the code blocklength, e.g. a received word with two errors in a (4,1) coding scheme.

In this system, using repetition codes of lengths from 1 up to *N*, the erase probability is calculated by:

$$P_\delta = P_2\binom{1}{2}\varepsilon(1-\varepsilon) + P_4\binom{2}{4}\varepsilon^2(1-\varepsilon)^2 +$$
$$+P_6\binom{3}{6}\varepsilon^3(1-\varepsilon)^3 + \cdots \quad (5)$$

Simplifying:

$$P_\delta = \sum_{n=1}^{N} ((n-1)mod\ 2)P_n \binom{n}{n/2} \varepsilon^{n/2}(1-\varepsilon)^{n/2}, \quad (6)$$

The proposed technique consists in the design of a coding scheme, such that, giving the maximum acceptable error probability and the maximum tolerate erase probability, the maximum transmission rate can be achieved. The solution of this optimization problem can be carried out by standard linear programming [11].

III. SIMULATIONS

In order to analyze the behavior of the coding technique in different scenarios, we used a variety of values for P_ε, P_δ and ε. It was also observed the influence of the value of *N* on the probability distribution.

A. The Erase Probability

During the simulations the following values were set: P_δ: 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}, 10^{-8}, 10^{-9} and 10^{-10}. The results of these simulations are illustrated in Fig. 1. The different curves correspond to different values of P_ε. The purpose is to inspect the influence of the probability P_δ on the transmission rate, when both P_ε and *N* have fixed values. It is observed that different values of P_δ have little influence on the channel transmission rate (*R*).

Table I shows the ten first probabilities of occurrence of the first repetition codes, namely *n*=1,2,…,10, calculated by assuming two different values for the erase probability P_δ, with *N*=100, $P_\varepsilon = 10^{-7}$ and $\varepsilon = 10^{-3}$, using 15 digit precision.

B. Adjusting the maximum repetition (N)

It was also analyzed the influence of the maximum chosen blocklength (*N*). Due to the large range of values for P_n., one can observe that the influence of *N* can be made very small, provided that this value is bigger than the number of repetition codes that have no negligible probabilities to be selected.

The tests showed that when it was adopted a 5-digit precision instead of 15 digits for P_n values, the absolute error in the transmission rate was less than 0.01%. For example, given *N*=10, $P_\varepsilon = 10^{-10}$, $P_\delta = 10^{-10}$ and $\varepsilon = 10^{-3}$, the transmission rate with 15-digit precision and *N*=100 is R= 14,3350896298743%. Choosing a 5-digit precision for P_n and the same parameters, only three codes non-zero probabilities of selection, and the new calculation of *R* gives *R*=14,3353333333333%. These values of P_n are shown in Table II.

C. Adjusting the maximum acceptable error probability (P_ε)

As observed in Equation 3, P_ε has a significant influence on the probability distribution, consequently on the transmission rate. Fig. 2 illustrates how *R* varies when different values of P_ε are adopted.

TABLE I. COMPUTED VALUES OF P_N FOR DIFFERENT VALUES OF P_δ

n	P_n ($P_\delta = 10^{-6}$)	P_n ($P_\delta = 10^{-7}$)
1	4.10410e-16	1.496e-14
2	2.467e-16	8.386e-14
3	0.0305	0.0301
4	0.167	0.0167
5	0.803	0.953
6	4.205e-16	4.375e-13
7	6.371e-16	2.819e-13
8	5.710e-16	2.224e-13
9	5.140e-16	1.913e-13
10	4.725e-16	1.721e-13

D. Channel error probability

The last inspected parameter was the value of the channel error probability, given fixed values of P_ε, *N* and P_δ. The result is presented in Fig. 4 ($P_\varepsilon = 10^{-10}$, $P_\delta = 10^{-6}$ and *N*=10). It can be observed that the coding is better suited for more reliable channels, e.g., for $P_\varepsilon = 5.10^{-9}$ and $P_\delta = 10^{-9}$ and the adaptive coding scheme is tested over a channel with $\varepsilon = 10^{-8}$, the resulting transmission rate is *R*=44,17%. Conversely, if we maintain a similar ratio between probabilities, but choose

P_ε=5.10^{-4} and P_δ =10^{-4} over a channel with ε=10^{-3}, the transmission rate is now given by R=44,02%.

TABLE II. COMPUTED VALUES P_N FRO DIFFERENT VALUES OF N

n	P_n (*N*=10)	P_n (*N*=100)
1	6.16e-18	9.931e-19
2	1.76e-17	2.966e-18
3	4.25e-17	2.113e-18
4	1.05e-16	5.667e-18
5	0.00655	0.00655
6	0.00501	0.00501
7	0.988	0.988
8	3.490e-14	1.067e-15
9	1.963e-14	6.00e-16
10	1.451e-14	4.439e-16

IV. WIMEDIA PAYLOAD PROTECTION VIA ADAPTIVE REPETITION CODES

As described in [10], the protection of WiMedia header exhibits better performance when using repetition codes instead of Reed-Solomon codes [5, 12]. As a consequence, it was offered the idea of replacing the coding scheme, but maintaining the compatibility (Fig. 3). The proposed method consists in repeating one of the branches of the convolutional code using a (3,1) code, thus resulting in 8 free bits to repeat some other bits from the other branches to preserve the length of the header, i.e., 600 bits (Fig. 4).

Our proposal is using those 8-bits to signalize the occurrence and/or the profile of the adaptive repetition scheme, making it possible to adjust the transmission rate accordingly to the distance between transmitter and receiver or environmental noise. In [10] it is also proposed to reduce the transmission rate from 1/3 to 1/4 in order to double the communication range. With different selected profiles it is possible to fine-tune the transmission rate.

These different profiles can be built from pre-selected probability distributions P_n. This idea is especially interesting for fixed-size frames. It allows reducing the computational complexity, thus requiring less processing time.

Both the transmitter and receiver *must* know the blocklength of the repetition code used for each transmitted bit. Since the theoretical computing of the probability distribution over the repetition codes ensemble is carried out *a priori,* it can be proposed a pseudo-random scheme to replace the random selection of repetition codes, which dramatically simplify channel coding.

V. CONCLUSIONS

A new strategy for error protecting of the WiMedia payload and header was investigated, which is particularly attractive due to its low complexity. It also provides great flexibility by offering a wide variety of coding schemes to be selected according to the transmission scenario. Further adaptive schemes have been recently proposed in the literature [13].

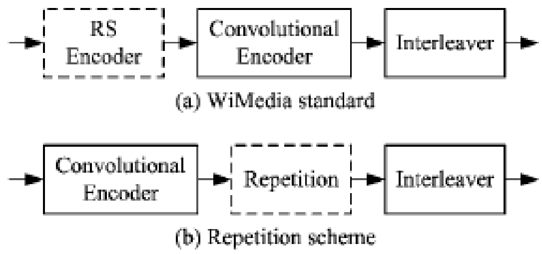

Fig. 4. Possible Coding schemes for the header transmission blocks in WiMedia UWB.

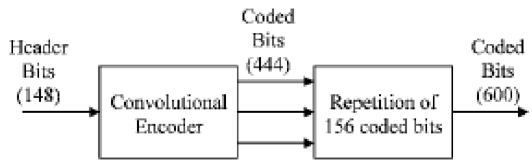

Fig. 5. Convolutional code used in the header of the WiMedia UWB.

Despite the (adaptive) repetition code still imply in reducing the transmission rate when it is required an error probability much lesser than the channel crossover probability, the investigated encoding scheme offers further flexibility to the standard repetition code, characterized by simplicity and low complexity. Additional investigation is currently being carried out, which includes performance assessment over different channels, such as the Gaussian channel.

REFERENCES

[1] R.G. Gallager, "Information Theory and Reliable Communication", NY:Wiley,1968.

[2] N.J.A. Sloane, A.D. Wyner, "Claude Elwood Shannon, Collected Papers", IEEE Press, 1993.

[3] J.T. Coffey and R.M. Goodman, Any Code of which we Cannot Think is Good, *IEEE Trans. Info. Theory*, IT 36, pp.1453-1461, 1990.

[4] S. Lin, D. Costello, M. Miller, Automatic-repeat-request error-control schemes, *IEEE Comm. Mag.*, Dec. Vol. 22, n.12,pp.5- 17, 1984

[5] S.B. Wicker, Error "Control Systems for Digital Commun. and Storage", Prentice Hall, NJ, 1995.

[6] G. Battail, Décodage ponderée Optimal des codes linéaires en blocks. I. Emploi simplifié du diagramme du treillis, *Ann. Télécomm*., 38, n.11-12, 1983, pp.443-459.

[7] J.K. Wolf, Efficient Maximum Likelihood Decoding of Linear Block Codes using a Trelles, *IEEE Trans. Info. Theory*, IT 24,n.1, 1978,pp.76-80.

[8] WiMedia Alliance, ECMA-368, "High Rate Ultra Wideband PHY and MAC Standard", 1st Edition, December 2005.

[9] H. Ghobad, "Wimedia UWB For W-USB and Bluetooth", John Wiley, 2008.

[10] R. Chrabieh and K. Akhavan, "Reed Solomon Code vs. Repetition in WiMedia UWB", 2007. *IEEE International Conference on Ultra-Wideband*, 2007. ICUWB 2007, pp. 669-673.

[11] D.G. Luenberger, "Linear and Nonlinear Programming", 2nd Ed., Springer, 2003 (491 p.).

[12] S.B. Wicker, "Adaptive Rate Coding Using Reed-Solomon Codes in a Hibrid-ARQ Protocol", ISSSE'89, Int. Symp. on Signals, Systems and Electronics, Erlang, FRG, pp.642-645, 1989.

[13] R.A. Saeed, N.C. Kyun, H. Mohamas, B.M. Ali, Hybrid automatic repeat request for WiMAX-mobile multi-hop relay using co-channel cancellation, Dec., pp.1-6, *Asia-Pacific Conference on Apllied Electromagnetics*, 2007.

APPENDIX

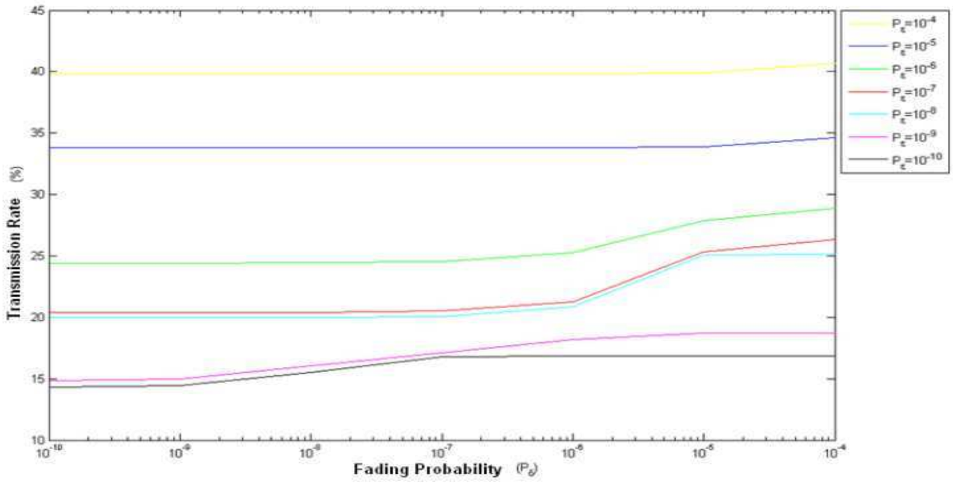

Fig. 1 – Maximum achievable transmission rate as a function of the acceptable erase probability.

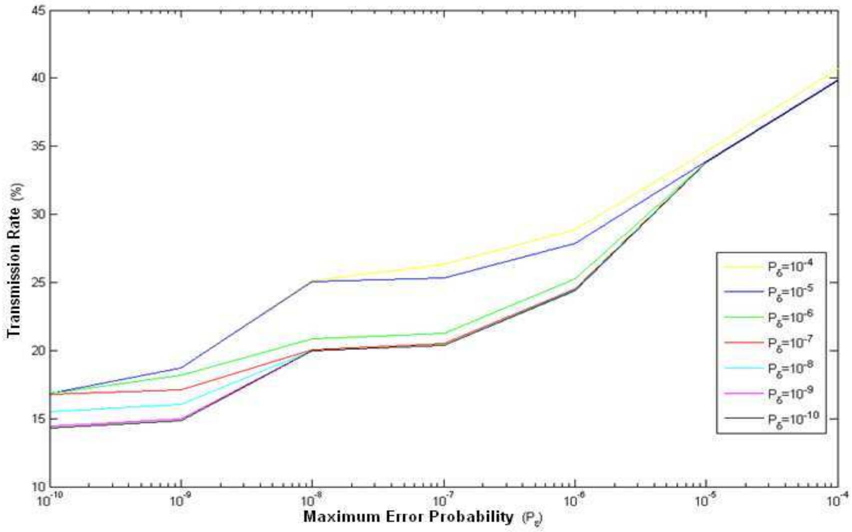

Fig. 2 – Maximum achievable transmission rate as a function of the acceptable error probability.

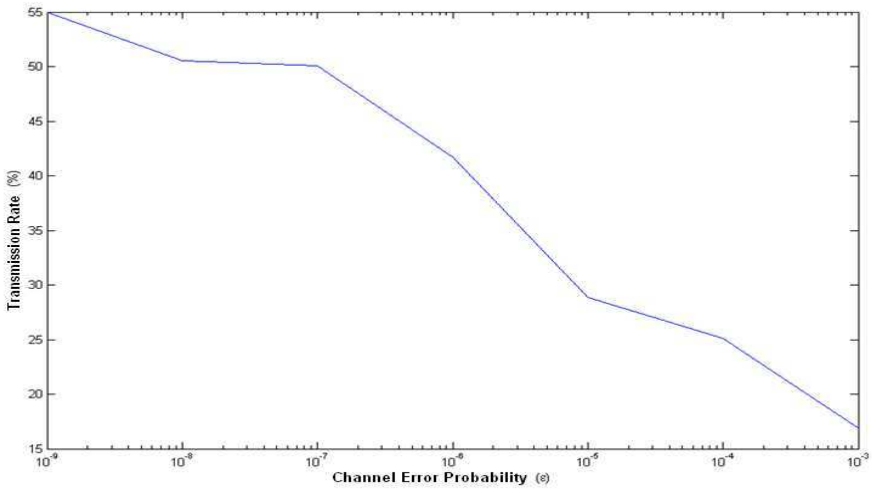

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Fig. 3. Achievable transmission rate as a function of the BSC crossover probability.